

AD-A251 270



## MENTATION PAGE

Form Approved  
OMB No. 0704-0188

Estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this burden to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Avenue, the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

REPORT DATE

3. REPORT TYPE AND DATES COVERED

FINAL 31 Aug 87 - 30 Sep 91

## 4. TITLE AND SUBTITLE

"AN INTEGRATED RESEARCH PROGRAM FOR THE MODELING,  
ANALYSIS & CONTROL OF AEROSPACE SYSTEMS" (U)

## 5. FUNDING NUMBERS

62301E

6121/00/DARPA

## 6. AUTHOR(S)

DRS. JA BURNS, EM CLIFF &amp; TL HERDMAN

## 7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Virginia Polytechnic Institute & State University  
Department of Mathematics  
Blacksburg VA 24061

AFOSR-TR-

8. PERFORMING ORGANIZATION  
REPORT NUMBER

92 0285

## 9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

AFOSR/NM  
Bldg 410  
Bolling AFB DC 20332-6448

10. SPONSORING/MONITORING  
AGENCY REPORT NUMBER

F49620-87-C-0116

## 11. SUPPLEMENTARY NOTES

DTIC  
S ELECTE D  
MAY 26 1992  
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## 12a. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for Public Release;  
Distribution Unlimited

## 12b. DISTRIBUTION CODE

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## 13. ABSTRACT (Maximum 200 words)

The following is a summary list of accomplishments under this contract: (1) ICAM was established in August of 1987. (2) An interactive computer system was developed. (3) Over 85 research papers were produced. (4) Over 33 students were supported in part by the contract. (5) More than 12 PhD's and 5 MS students were produced. (6) Seven Post Doctoral Associates were supported. (7) More than 65 short term visitors came to Virginia Tech (16 of these were long term visitors). (8) Over 100 scientists attended the conference on Numerical Methods for Partial Differential Equations.

## 14. SUBJECT TERMS

15. NUMBER OF PAGES  
28

16. PRICE CODE

17. SECURITY CLASSIFICATION  
OF REPORT

UNCLASSIFIED

18. SECURITY CLASSIFICATION  
OF THIS PAGE

UNCLASSIFIED

19. SECURITY CLASSIFICATION  
OF ABSTRACT

UNCLASSIFIED

## 20. LIMITATION OF ABSTRACT

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**FINAL REPORT**

on

**AN INTEGRATED RESEARCH PROGRAM FOR THE  
MODELING, ANALYSIS AND CONTROL OF AEROSPACE  
SYSTEMS**

**(Contract No. F49620-87-C-0116)**

for

**1 September 1987 - 30 September 1991**

by


**J. A. Burns  
Department of Mathematics**

**E. M. Cliff  
Department of Aerospace and Ocean Engineering**

**T. L. Herdman  
Department of Mathematics**

**Virginia Polytechnic Institute & State University  
Blacksburg, Virginia 24061**

**March 3, 1992**

**92-13049**  


**92 5 15 020**

March 6, 1992  
12:42 PM

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Availability Codes	
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A-1	

## I. PREFACE

This FINAL REPORT report contains a summary of the activities supported under the DARPA Contract F49620-87-C-0116 during the period from September 1, 1987 to September 30, 1991. This project is concerned with the development of an integrated research program for the modeling, analysis and control of aerospace systems. In addition to the usual goal of production of research papers, the project was concerned with the training of new mathematical scientists and engineers and the coordination of an active exchange program between scientists.

The principal investigators were Professors John A. Burns, Department of Mathematics, Eugene M. Cliff, Aerospace and Ocean Engineering and Terry L. Herdman, Department of Mathematics.

The proposed statement of work contained three essential items:

A) The establishment of an interdisciplinary center for the theoretical and computational study of innovative control technology for aerospace vehicles and flexible structures.

B) The development of a research program in modelling, analysis and control of aerospace systems. This aspect of the project involved the development of new computational algorithms for identification, control and simulation of dynamical systems that govern fluid flows, structural vibrations and flight mechanics.

C) The coordination of an active exchange program between graduate students and various scientists and engineers.

The Interdisciplinary Center for Applied Mathematics (ICAM) was formed in August 1987 to promote and facilitate interdisciplinary research and graduate education in applied mathematics at Virginia Tech. This center has provided the mechanism needed to promote the studies of control technology for aerospace vehicles and flexible structures. In addition, ICAM helped organize the active visitors program and the conference on numerical partial differential equations.

## II. BRIEF SUMMARY OF ACCOMPLISHMENTS

The following list contains a summary of the various activities and accomplishments that were funded in part by this contract. A detailed description of the research projects, personnel supported, students produced and other activities are presented in the following sections.

ICAM was established in August of 1987

Developed an interactive computer system

Over 85 research papers were produced

Over 33 students were supported in part by this grant

More than 12 Ph.D.s and 5 M.S. students were produced

7 Post Doctoral Associates were supported

More than 65 short term visitors came to Virginia Tech

16 of these were long term visitors

Over 100 scientists attended the conference on  
Numerical Methods for Partial Differential Equations

### III. DESCRIPTION OF PROJECT

#### COMPUTING FACILITIES

The local computing environment is built around a DEC GPX-II Graphics Workstation. The GPX-II with three 159 MB hard disks performs a file server function on a local Ethernet. A VAX 2000 with a 70 MB hard disk and several personal computers are connected to the GPX-II via an Ethernet. A LNO3 laser printer acts as the printing device for the system. The system runs under DEC Ultrix; we have installed the GKS graphics system and language compilers (FORTRAN and C). The DELIGHT.MIMO software, which links a sophisticated non-smooth optimization package to some linear system software, is on the system. The package was kindly furnished by Professor E. Polak, Electrical and Computer Engineering Department, University of California-Berkeley.

#### RESEARCH

The basic research goal of this project is the study of control designs for hybrid aerospace systems. The procedure for designing high performance controllers for complex aerospace systems includes:

1. Developing accurate mathematical models of the physical problem, and
2. Constructing computational algorithms specifically suited for control design and analysis.

Step 2 is needed since computational algorithms developed for simulation may not be suitable for control design and optimization. This problem occurs, for example, when the governing equations are coupled partial functional differential equations of mixed type. Our approach is to combine fluid/structure dynamics and control in one model.

This approach leads to models of the form:

$$\dot{z}(t) = Az(t) + Ly(t) + Bu(t)$$

$$\dot{\sigma}(t) = F\sigma(t) + Gz(t)$$

$$y(t) = C\sigma(t) + Dz(t)$$

where the parameters A, L, B, F, G, C, and D are partial functional or ordinary differential operators. For example,

A might be a partial differential operator which describes the elasticity of the structure while F might be a quasi-linear partial differential operator which describes the fluid dynamics. The rest of the parameters govern the coupling between fluid and structural dynamics and are highly dependent on the geometry of the structure. In the next few pages we describe some of the projects supported in part under this contract.

### AEROELASTIC MODELING

For the past four years we have been studying issues related to the design of high performance controllers for complex aerospace systems in which the elastic motions of the structure are coupled with the motions of the surrounding fluid. Typically, the equations of motion of such systems involve coupled differential and singular integral equations, requiring a considerable amount of mathematical analysis to provide the necessary dynamic modeling, estimation, computation and control. In particular, we have studied the flutter problem for a three-degrees-of-freedom "typical" airfoil section in two-dimensional, incompressible flow (the so-called Theodorsen problem) where the dynamic model describing the composite system can be formulated as a system of singular neutral functional differential equations. Motivated by this model we have addressed the following questions: i) (Well-Posedness) Under what conditions can singular neutral equations be considered in the framework of linear infinite-dimensional systems? ii) (Approximation) Using the abstract approximation framework available for linear infinite-dimensional systems (i.e., Trotter-Kato type theorems) how can we construct robust, highly accurate computational schemes for the control and identification of systems governed by singular neutral functional differential equations?

Our study of the well-posedness of functional differential equations, including the aeroelastic system discussed above, has been in the context of functional analytic semigroup theory. The applicability of the semigroup approach for the development of approximation techniques for parameter estimation and optimal control for such system requires the identification of an appropriate state-space which not only yields the  $C_0$ -semigroup but also allows one to obtain a dissipative estimate for the infinitesimal generator for the semigroup. This requirement is due to the convergence of the numerical schemes being established by using the Trotter-Kato semigroup approximation theorem. This dissipative estimate for the aeroelastic system with state-space  $R^8 \times L_2$  is guaranteed, however, finding the equivalent norm on that space, which is needed to give the dissipative estimate, is not

straightforward. For retarded functional differential equations and atomic neutral functional differential equations on the state space  $R^n \times L_{2,g}$ ,  $g$  a weight function, a well-developed theory is available. However, for singular neutral systems, such as our aeroelastic system, a characterization of  $g$  that is needed to obtain the dissipative estimate was unknown. Our investigation shows that the selection of  $g$  plays a key role in the development of the semigroup formulation for our system.

We have established that the finite delay version of the aeroelastic systems generates a  $C_0$ -semigroup on the product space  $R^7 \times L_{2,g}$  and showed that the infinitesimal generator of that semigroup does, indeed, satisfy a dissipative estimate. Here the function  $g$  is defined by  $g(s) = [1 + (-s)^{-1}]^{\frac{1}{2}}$ . This framework was extended to include more general singular neutral functional differential equations. In the general case we allow a weight function  $g$  satisfying (i)  $g(s) > 0$ ,  $g(\cdot) \in L_1$ ,  $\dot{g} \geq 0$  and (ii) for any given  $c > 0$ , there is a  $\tau > 0$  such that  $g(s) \geq cg(s)$  on  $[-\tau, 0]$ . We developed an approximation scheme and applied it to two examples to illustrate examples to indicate the feasibility of our approach. One example is a scalar equation with inconsistent initial data, no smoothness on the solution at  $t = 0$ , the other example is a two-dimensional system which exhibits characteristics similar to those one observes in the aeroelastic system.

We have completed an in-depth study of the derivation of the equations for the aeroelastic system assuming that the circulation history belongs to a weighted  $L_2$  space. We established that the assumption that the "past history" of the derivation of the total circulation function belongs to  $L_{2,g}$ , same  $g$  that provided the dissipative estimate, is sufficient to assure the validity of the Söngen's inversion formula for the solution of the airfoil equation. Therefore, we have obtained compatibility of the validity of the inversion formula and the well-posedness of the resulting neutral system. The resulting state space  $R^7 \times L_{2,g}$  and the numerical techniques described above provides a suitable setting for control design for the aeroelastic system.

For the complete system described above the evolution equation for the circulation on the airfoil was coupled to the rigid-body dynamics of the airfoil to obtain the infinite delay singular neutral system of the form

$$\frac{d}{dt} \left[ Ax(t) + \int_{-\infty}^0 A(s)x(t+s)ds \right] = Bx(t) + \int_{-\infty}^0 B(s)x(t+s)ds + f(t)$$



as the mathematical model. The  $8 \times 8$  matrix  $A$  is singular, the  $8 \times 8$  matrix function  $A(s)$  is weakly singular,  $A_{gg}(s) = ((Us - 2)/Us)^{\frac{1}{2}}$  and  $f$  represents a forcing term (possible control) for the system. The state of the system includes the past history of  $\dot{\Gamma}$  where  $\Gamma$  denotes the total airfoil circulation. This past history of  $\dot{\Gamma}$  may not be observable for the entire past time  $(-\infty, 0)$ . However, one would be able to observe this past history over a finite time interval say  $[-r, 0]$ . Since the kernel function  $A(s)$  is not integrable on  $(-\infty, 0)$  it is not possible to address the infinite delay problem as a finite delay problem by mapping the interval  $(-\infty, 0)$  to some finite interval  $[-T, 0]$ . On the other hand the "finite delay" version

$$\frac{d}{dt} \left[ Ax(t) + \int_{-r}^0 A(s)x(t+s)ds \right] = Bx(t) + \int_{-r}^0 B(s)x(t+s)ds + f(t)$$

has been extensively studied. During this period we have investigated the possibility of taking advantage of our results for the finite delay system to study the infinite delay system. We developed an approximation technique for the infinite delay system on an interval of the form  $[0, T]$  using the approximation techniques we developed for the finite delay system. In order to accomplish this we have made use of the special structure found in the aeroelastic system. In particular, the representation of the aeroelastic system allows one to view the system as having an integrodifferential Volterra component and a singular integral component. We have shown that the infinite delay singular integral component can be approximated using the corresponding finite delay singular integral component. Our results include error estimates for the approximation. Our two step approximation procedure starts with truncating the infinite delay term. The approximation is dependent on the specified initial data. The desired error bound together with the time interval  $[0, T]$  on which one wishes to find the solution dictates the necessary truncation. The second step consists of employing the approximation scheme established for the associate finite delay system.

#### PARAMETER IDENTIFICATION

It has been shown that fractional order operators can provide accurate models for viscoelasticity damped structures. The governing equation for such systems has the

form of a partial-integral equation. These systems can be viewed as neutral functional differential equations having a structure common to the mathematical models described above for aeroelastical systems. For our study of viscoelastic systems we focused our attention to the task of developing a parameter identification scheme for these systems. As a first step, we investigated the effectiveness of quasilinearization for parameter identification in both linear and nonlinear parabolic partial differential equations. Our study included the heat equation where the unknown parameter was the spatially varying diffusion coefficient and the nonlinear Burgers' equation.

During this period we developed a scheme for identification of unknown parameters in parabolic partial differential equations. Our study included theoretical detail and careful numerical testing. We established the Fréchet differentiability of the solution with respect to the unknown parameter which was shown to be a primary result necessary to ensure local convergence of the approximation scheme used to find the unknown parameter. Our numerical examples that we present demonstrate the rapid convergence and accuracy of the algorithm. Our efforts to date involving the use of quasilinearization in nonlinear partial differential equations have only been numerical ones. However, the success of our numerical testing leads us to believe that our quasilinearization scheme will provide an effective approach to parameter identification for certain nonlinear equations.

#### CONTROL ALGORITHMS FOR INFINITE DIMENSIONAL SYSTEMS

The development of practical control algorithms for infinite dimensional systems described by partial differential equations requires that approximations be introduced at some point in the analysis and design. A typical engineering approach to such problems is to first introduce some type of finite dimensional "design model" and then use this approximate model to design the control. Another approach to this problem involves the use of "truncated balanced realizations" to construct the finite dimensional design models. We compared and investigated these two approaches on several control problems governed by partial differential equations. We discovered an interesting, and somewhat surprising, result. When the two approaches were applied to a problem in the control of a thermal processes, it was found that the standard finite element scheme was an order of magnitude more robust than the "truncated balance realization" method. Moreover, the balanced finite element model produced a pre-conditioned system that was extremely robust. These discoveries have considerable impact on the conditioning of the numerical algorithms used in control design. To be useful, however,

it is essential that one have a practical method for computing these condition numbers. We have discovered that certain Galerkin methods are more robust than the "standard" methods often used in engineering practice.

### ACTIVE CONTROL OF SMART MATERIALS

During the past ten years considerable progress has been made in the development and understanding of new and complex materials. These materials are the essential building blocks and an enabling technology for many current and future DOD missions. Shape memory alloys, piezoelectric materials and fiber optics are being used to develop new actuators and sensors for active control of "smart structures" and for structural-acoustics interaction problems. There are numerous DOD applications of such technologies. Although considerable progress has been made in the area of material science, many of these breakthroughs are based on experimental and simulation studies. There is a tremendous void in the development of useful models and corresponding practical computational algorithms for control design of such systems. The fact that the construction of practical computational algorithms for control of these new materials is lagging behind the development of experimental and simulation tools could severely limit the scope of future DOD missions. We addressed the problem of developing practical, fast and robust computational algorithms for the identification of systems involving these new materials. All of the computational algorithms were based on mathematical models that accurately reflect the physics of the systems.

We initiated an investigation into the use of these shape memory alloys as actuators in active control designs. These alloys are best described by thermo-mechanical models consisting of coupled (and nonlinear) hyperbolic and parabolic partial differential equations. The development of computational algorithms for designing controllers for such systems is an immensely complex problem and the subject of several ongoing research projects. In addition to the obvious difficulties related to the nonlinearities, the basic thermo-elastic coupling often gives rise to nonstandard mathematical models and leads to several problems in developing computational algorithms for control. Therefore, the computational methods for controlling a linear thermo-elastic system may be viewed as a first step toward the ultimate nonlinear problem. With this motivation in mind, we considered the problem of controlling a class of coupled partial differential equations that describe the linearized motions of a thermo-mechanical structure. The basic approach was to combine approximation theory with state space modelling to develop convergent computational

algorithms for LQR control designs. Our results were extremely encouraging.

### OPTIMAL MANEUVERS

There is considerable interest in increased agility for fighter aircraft. The belief is that all-aspect missiles put a premium on a first-shot capability. One mechanism for increasing agility is the use of thrust-generated moments to control aircraft attitude. This is to be investigated with the DARPA-sponsored X-31A research vehicle. We conducted a study of optimal maneuvers with mathematical models of aircraft dynamics.

In particular, we have examined optimal rigid-body angular rate control via an approximate dynamic model, which admits analytical solutions of the optimality conditions. The analysis reveals that over a large range of boundary conditions, there are, in general, several distinct external solutions. Second-order necessary conditions have been investigated to establish local optimality of candidate minimizers. Global optimality of the external solutions has been studied.

We have also studied the optimal angular rate problem using an exact dynamic model. Numerical solutions of optimality conditions were obtained which corroborate and extend the findings of the approximate problem. The qualitative feature of multiple external solutions is retained and as before, certain external solutions do not satisfy the Jacobi necessary condition. The choice of minimizing solution has been narrowed down to two sub-families of extremals. A locus of Darboux points was obtained which demarcates the domains over which these two sub-families are globally minimal.

The above studies look at minimum control effort families of external solutions. As a next step, we examined the minimum time control of angular rates, with prescribed hard bounds on available control. Existence of singular subarcs in time-optimal trajectories was explored. Qualitative features exhibited by the minimum-effort problem are preserved. In addition, the control space was deformed to allow roll control and its effect on external solutions was investigated.

In the final phase, we introduced kinematics into the optimal control problem. Minimum time attitude control of a rigid-body were investigated with prescribed hard bounds on available control. A preliminary numerical survey of first-order necessary conditions reveals that there are several distinct external solutions. The character of external solutions depends on whether the pitch or the yaw motion

assumes the dominating role in controlling the roll motion. Moreover, certain spatial symmetries have been identified. Specific maneuvers such as: roll around the velocity vector, and fuselage pointing have been analyzed.

#### FLUID FLOW CONTROL

We have made considerable progress on a simple fluid flow control problem. The problem is concerned with the control of Burgers' Equation. We have extended some of the basic finite dimensional ideas to this non-linear partial differential equation model. Burgers' Equation is an excellent test problem. We have applied various state feedback schemes (including "linearization followed by standard LQG theory") and investigated the properties of the corresponding non-linear closed-loop systems. The major tasks here were the development of fast computational algorithms and the analysis of the closed-loop system. As to be expected, several numerical difficulties occur in problems with high Renold's number. All of these issues were addressed and feedback laws were computed that not only enhanced stability, but reduced shock formation.

#### IV. LABORATORY INTERACTION

In addition to the research described above, we had considerable interaction with Laboratory personnel at several laboratories including:

- \* Air Force Astronautics Laboratory, Edwards Air Force Base;
  - Dr. Alok Das
  - Capt. Doug DeHart
- \* Air Force Armament Laboratory, Eglin Air Force Base;
  - Dr. J. Cloutier
- \* NASA Langley Research Center;
  - Mr. J. Elliott
  - Dr. C. Gracey
  - Dr. R. Ou
- \* Wright Laboratory (FIGC), Wright-Patterson Air Force Base;
  - Mr. Jerry Jenkins
  - Mr. Charles Suchomel
- \* Wright Laboratory (FIBG), Wright-Patterson Air Force Base:
  - Mr. Jon Lee
- \* Wright Laboratory (FFMG)
  - Mr. Tom Cord
  - Mr. Kevin Langen

We saw this interaction as an important part of our research and educational program.

## V. VISITORS PROGRAM

The research program was significantly enhanced by the outside visitors that helped create conditions for interactions among mathematicians, scientists, engineers and computer scientists. During the period of this contract we provided partial support for many of the more than 65 short term visitors. These visits have already produced several joint papers and precipitated many new interdisciplinary research problems.

### LIST OF VISITORS

#### LONG TERM VISITORS

H.T. Banks - University of Southern California  
 Dennis W. Brewer - University of Arkansas  
 Martin Brokate - University of Kaiserslautern  
 Ingo Eichenseher - University of Augsburg, Germany  
 Richard H. Fabiano, Jr. - Brown University  
 Mitchell Feigenbaum - Rockefeller University  
 Elena Fernandez - Instituto de Desarrollo Tecnológico, para la Industria Química  
 Wilfred M. Greenlee - University of Arizona  
 Israel Gohberg - Tel Aviv University  
 Karl H. Hoffmann - University of Augsburg  
 Franz Kappel - University of Graz  
 Gunther H. Peichl - University of Graz  
 Dominique Pelletier - Ecole Polytechnique of Montreal  
 Janos Turi - University of Texas at Dallas  
 George Weiss - Brown University  
 Klaus H. Well - Deutsche Forschungs- und Versuchsanstalt für Luft- und Raumfahrt e.V., Oberpfaffenhöfen, Germany.

#### SHORT TERM VISITORS

Juergen Ackermann - Institute of Flight Systems Dynamics  
 Richard Babarsky - James Madison University  
 Ivo Babuska - University of Maryland  
 J.P. Boris - US Naval Research Laboratory  
 M. Breitner - Technical University of Munich and German Aerospace Research Establishment  
 T. J. Bridges - Worcester Polytechnic Institute  
 Roger Brockett - Harvard University  
 Roland Bulirsch - Technical University of Munich  
 Peter Castro - Eastman Kodak, Rochester, NY  
 Kenneth L. Cooke - Pomona College  
 John Dennis - Rice University

Richard E. Ewing - University of Wyoming  
 Robert E. Fennell - Clemson University  
 George J. Fix - University of Texas Arlington  
 David Fox - University of Minnesota  
 Avner Friedman - University of Minnesota  
 Revaz Gamkrelidze - Moscow, Russia  
 James G. Glimm - State University of New York at Stony Brook  
 Werner Grimm - Deutsche Forschungs- und Versuchsanstalt für Luft- und Raumfahrt e.V., Oberpfaffenhöfen, Germany.  
 Jack Hale - Georgia Tech  
 James Holloway, University of Michigan  
 Kazufumi Ito - Brown University  
 Elizabeth Jessup - Knoxville, TN  
 Franz Kappel - Universität Graz  
 R. Baker Kearfott - University of Southwestern Louisiana  
 C. Tim Kelley - North Carolina State University  
 Belinda King - Clemson University  
 Art Krener - University of California at Davis  
 Irena Lasiecka - University of Virginia  
 Alan J. Laub - University of California-Santa Barbara  
 Zhang Liu - University of Minnesota  
 Dahlard L. Lukes - University of Virginia  
 John Lund - Montana State University  
 E. F. Mischenko - Moscow, Russia  
 J. D. Murray - Oxford University  
 Stephen G. Nash - George Mason University  
 R. A. Nicolaides - Carnegie Mellon University  
 Stanley J. Osher - University of California-Los Angeles  
 Gunther H. Peichl - Universität Graz  
 Robert Pego - University of Maryland  
 Dominique Pelletier - University of Montreal  
 C. Peskin - Courant Institute, New York University  
 Samuel M. Rankin, III - Worcester Polytechnic Institute  
 M. Reed - Duke University  
 Gary Rosen - University of Southern California  
 Ekkhard Sachs - North Carolina State University  
 Ricardo Sanchez-Pena - University of Buenos Aires  
 Lee A. Segel - Weizmann Institute of Science, Rehovot, Israel  
 George Sell - Army High Performance Computing Research Center, Minneapolis, Minnesota  
 James Sochacki - ISC, LaRamie, Wyoming  
 Felipe de Souza - Instituto Tecnológico de Aeronautica  
 Jason L. Speyer - University of Texas  
 Eitan Tadmor - Tel Aviv University  
 Roberto Triggiani - University of Virginia  
 Janos Turi - Worcester Polytechnic Institute  
 Homer F. Walker - Utah State University  
 Robert C. Ward - Oak Ridge National Laboratories  
 Joseph C. Watkins - University of Southern California  
 W. Wedig - University of Karlsruhe  
 Helena S. Wisniewski - Lockheed Corporation



Alastair D. Wood - Dublin City University, Dublin, Ireland  
Bostwick Wyman - Ohio State University  
David M. Young - University of Texas at Austin  
Songmu Zheng - University of Utah  
Enrike Zuazua - Courant Institute of Mathematical Sciences,  
New York University

## VI. OTHER ACTIVITIES

### CONFERENCE ON NUMERICAL SOLUTIONS OF PARTIAL DIFFERENTIAL EQUATIONS SEPTEMBER 24-27, 1988

One long term goal of this project is to develop theoretical and computational tools for attacking complex fluid/structural control problems. As an important step, we organized a conference on numerical solutions of partial differential equations. The goal of this conference was to bring together several experts in numerical analysis and control theory and have them present a survey of recent progress in their area of expertise. The conference attracted over 100 attendees. In addition to support from DARPA, partial support was obtained from the Air Force Office of Scientific Research, the Virginia Tech Department of Mathematics and the Interdisciplinary Center for Applied Mathematics.

#### Invited Speakers

I. Babuska - University of Maryland  
H. T. Banks - Brown University  
R. E. Ewing - University of Wyoming  
G. J. Fix - University of Texas Arlington  
J. G. Glimm - Courant Institute of Mathematical Sciences  
New York University  
R. A. Nicolaides - Carnegie Mellon University  
S. J. Osher - University of California Los Angeles  
D. L. Russell - University of Wisconsin/Virginia Polytechnic  
Institute and State University  
E. Tadmor - Tel Aviv University

#### Participants Supported

Robert Acar - University of Oklahoma  
Krishan M. Agrawal - Virginia State University  
Christoph Börgers - University of Michigan  
Moysey Brio - University of Arizona  
Christopher L. Cox - Clemson University  
M. Desai - Brown University  
Donald Estep - Georgia Tech  
Ben G. Fitzpatrick - Brown University  
Ali Hajjafar - The University of Akron  
S. I. Hariharan - The University of Akron  
Lisheng Hou - Carnegie Mellon University

Belinda B. King - Clemson University  
Kevin L. Kreider - Ames Laboratory  
John Lund - Montana State University  
Amnon J. Meir - Carnegie Mellon University  
Milan Miklavcic - Michigan State University  
Hae-Soo Oh - University of North Carolina Charlotte  
David Rebnord - Brown University  
Peter I. Reichmann - The Catholic University of America  
Hisham Sarsour - North Carolina State University  
Jingyou Sun - University of Texas Arlington  
Hillel Tal-Ezer - ICASE  
H. Tran - Brown University  
G. Wade - Brown University  
Junping Wang - Cornell University  
Yun Wang - Brown University  
Qi Yu Zhang - Carnegie Mellon University

## VII. PERSONNEL

### PRINCIPAL INVESTIGATORS

John A. Burns - Professor, Department of Mathematics  
 Eugene M. Cliff - Professor, Aerospace and Ocean Engineering  
 Terry L. Herdman - Professor, Department of Mathematics

### ASSOCIATE INVESTIGATORS

William T. Baumann, Electrical Engineering,  
 Yuriko Renardy, Department of Mathematics  
 David Russell, Department of Mathematics  
 Janos Turi, Department of Mathematics, University of Texas  
 at Dallas

### POST DOCTORAL FELLOWS

Joseph Z. Ben-Asher - Ph.D., Virginia Tech  
 Deborah Brandon - Ph.D., Carnegie Mellon  
 Wenceslao Cebuhar - Ph.D., Harvard  
 Guenter Leugering, Ph.D., Technische Hochschule Darmstadt  
 Robert E. Miller - Ph.D., Virginia Tech  
 Gunther Peichl - Ph.D. University of Graz  
 Moshen Tadi - Ph.D., Virginia Tech

### GRADUATE STUDENTS SUPPORTED

Joseph Ben-Asher--Ph.D. Aerospace Engineering, June 1988  
 Marwan Bikdash--Ph.D. Student/Electrical Engineering  
 Spiro Bocvarov--Ph.D. Student/Aerospace & Ocean Engineering  
 Philip Bushong--Ph.D. Student/Aerospace & Ocean Engineering  
 B. Charrier--M.S. Student/Aerospace & Ocean Engineering  
 Rajiv Chowdhry--Ph.D. Student/Aerospace & Ocean Engineering  
 Diane DeWalt--Ph.D. Student/Aerospace & Ocean Engineering  
 Patricia Hammer--Ph.D. Student/Mathematics  
 Scott Hansen--Ph.D. Student/Mathematics  
 David Hill--Ph.D. Student/Mathematics  
 Scott Inch--Ph.D. Student/Mathematics  
 Makoto Ishigaki--Ph.D. Student/Mathematics  
 Sungkwon Kang--Ph.D. Student/Mathematics  
 Ruta Kher--M.S. Student/Mathematics  
 Renjith Kumar--Ph.D. Student/Aerospace & Ocean Engineering  
 Gyou Bong Lee--Ph.D. Student/Mathematics  
 Hamadi Marrekchi--Ph.D. Student/Mathematics  
 Craig Martell--M.S. Student/Aerospace & Ocean Engineering  
 Kimberly Oates--M.S. Student/Mathematics

Robert Miller--Ph.D. Student/Mathematics  
 Richard Oleksuk--Ph.D. Student/Electrical Engineering  
 John Ong--Ph.D. Student/Mathematics  
 Marek Rakowski--Ph.D. Student/Mathematics  
 John Rogowsky--Ph.D. Student/Electrical Engineering  
 Reuben Spies--Ph.D. Student/Mathematics  
 Kiran Suwal--M.S. Student/Aerospace & Ocean Engineering  
 Mohsen Tadi--Ph.D. Student/Engineering Science & Mechanics  
 Brian Thompson--M. S. Student/Aerospace & Ocean Engineering  
 Pangiotas Tsiotras--Ph.D. Student/Aerospace & Ocean  
 Engineering  
 Gopal Vasudevan--Ph.D. Student/Aerospace & Ocean Engineering  
 Xiaohong Zhang--Ph.D. Student/Mathematics  
 Lan Zhang -- Ph.D. Student/Mathematics

### **Undergraduate Students**

Jennifer Moore--B.S. Student/Aerospace & Ocean Engineering

### **SUPPORT PERSONNEL**

Nancy D. Smith - Research Administrative Officer  
 Ginger Clayton - Research Administrative Officer  
 Debbie F. Journell - Research Administrative Officer  
 Kenneth P. Hinson - Computer Systems Senior Engineer  
 Brett M. Marcus - Programmer  
 Jennifer Moore - Programmer  
 Tina Burrell - Programmer  
 Christine Freeman - Office Specialist  
 Kenneth P. Hinson - Computer Systems Senior Engineer  
 Robin C. Endelman - Programmer  
 Alice Bell - Secretary  
 Michelle Boyd - Secretary

Ruta Kher--M.S. -1987, Mathematics  
Craig Martell--M.S.- 1991, Student/Aerospace & Ocean  
Engineering  
Kimberly Oates--M.S. - 1991, Mathematics  
Richard Oleksuk--M.S. - 1991, Electrical Engineering  
Kiran Suwal--M.S. - 1989, Aerospace & Ocean Engineering

## IX. PUBLICATIONS

The following papers were supported in part under this contract:

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20. S. Katzir, E. M. Cliff and H. J. Kelley, "Best-range  
study for a boost-sustain missile," Proc. 1988 American  
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